Muons and atomic spectroscopy

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Outline

Muonic hydrogen: Lamb shift, proton radius,
and the normal hydrogen Lamb shift

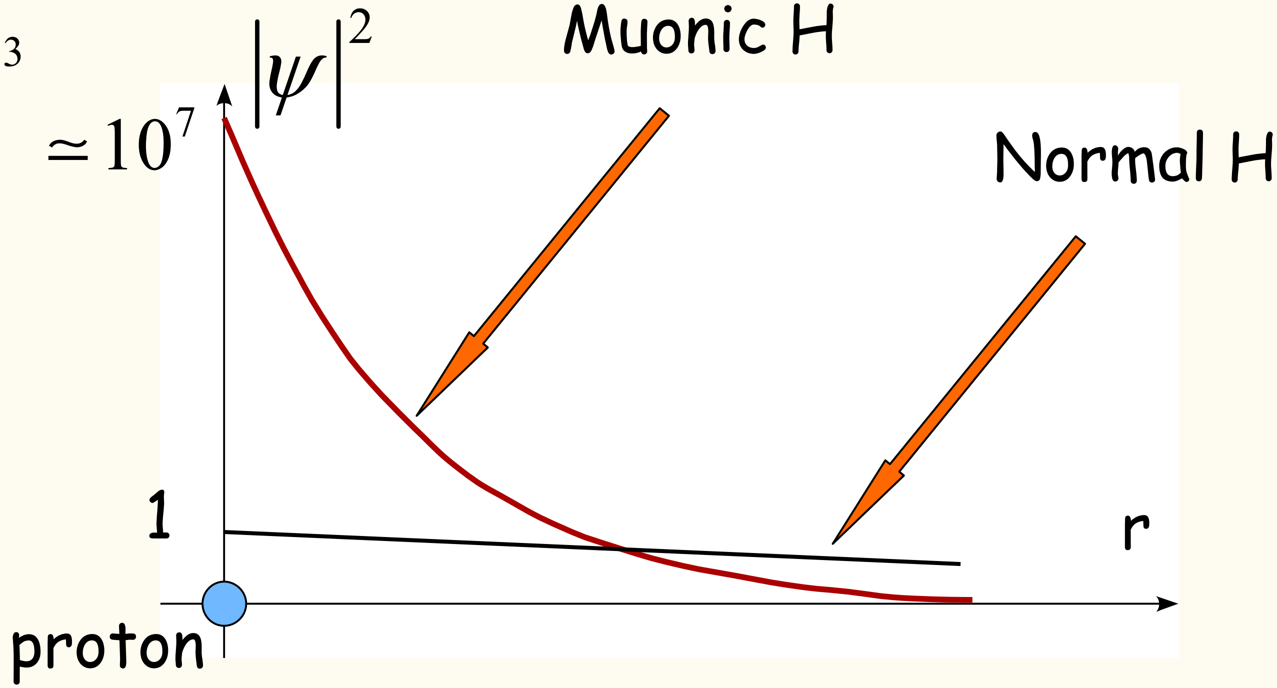
New theoretical and experimental developments

Muonium: muon magnetic moment and g-2

Outlook

Muonic hydrogen and the proton radius

$$\left(\frac{m_\mu}{m_e}\right)^3 \simeq 10^7$$



Sensitivity of energy levels to proton structure:

$$\frac{|\psi_\mu(0)|^2}{|\psi_e(0)|^2} = \left(\frac{m_\mu}{m_e}\right)^3 \simeq 10^7$$

How accurately can we compare theory and measurements?

At the time of the PSI proposal R-98-03.1 (1998)

Laser spectroscopy of the Lamb shift in muonic hydrogen

precision of the 1S-2S measurement in Hydrogen:

840 Hz $\rightarrow 3.5 \cdot 10^{-13}$

Now: $1.9 \cdot 10^{-14}$

Note: four orders of magnitude over 15 years; Nobel 2005

New theoretical tools:

- NRQED
- dimensional regularization
- asymptotic expansions
- symbolic computation

Proton has more than one radius…

Proton structure effect in Lamb shift:
electric charge radius,
PSI goal → 0.7 percent accuracy

Hyperfine splitting:
magnetic and electric distributions,
Zemach radius
Brodsky et al. (2005), 1.5%

State of Lamb shift theory

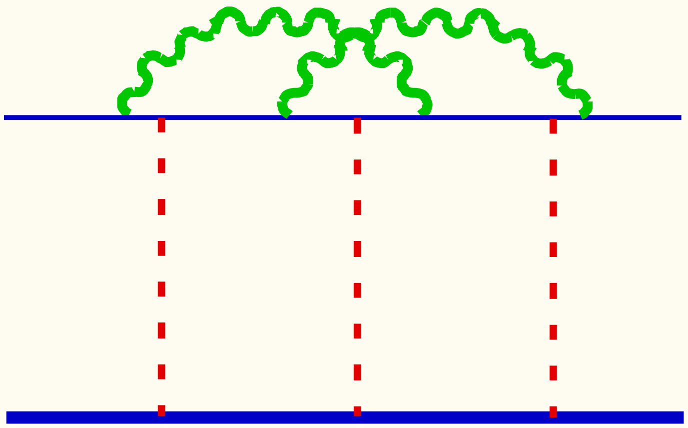
Experiment:
PRL (1999)
Schwob et al.

$$L_{1S} = 8172.837(22)\ \text{kHz}$$

Proton radius

Theory:
Pachucki
Pachucki & Jentschura
Yerokhin, Indelicato, Shabaev

$$L_{1S} = 8172.804(32)(4)\ \text{kHz}$$



Should such effects be studied? Will r_p be improved?

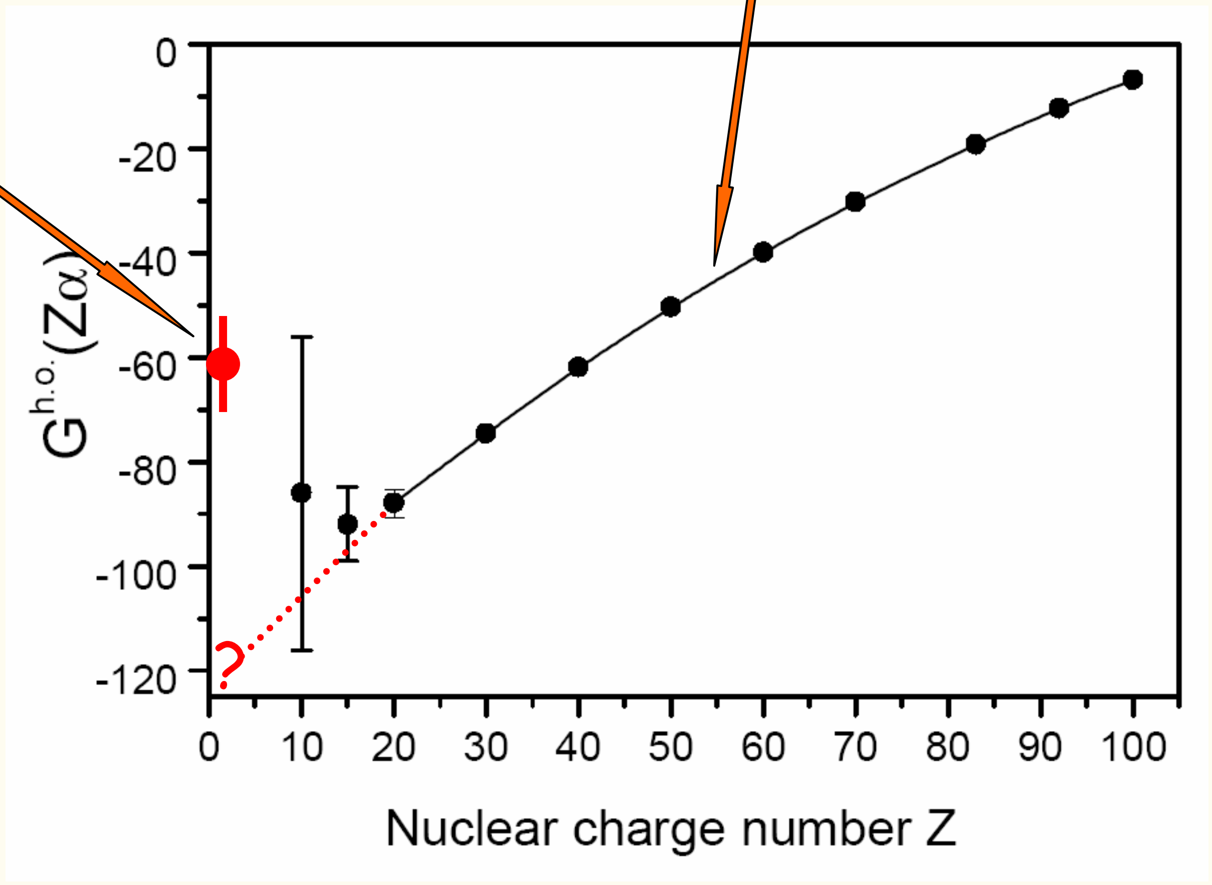
Challenges of the bound-state theory: two-loop self-energy

Nonperturbative study

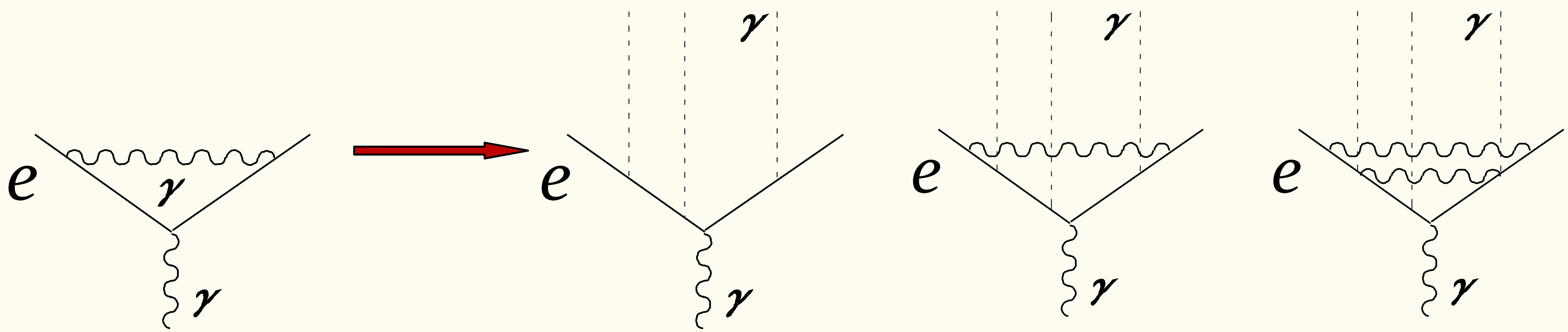
Yerokhin, Indelicato, Shabaev

Perturbative calculation

Pachucki & Jentschura



Two-loop bound-state calculations are possible: bound-electron g-2



$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6$$

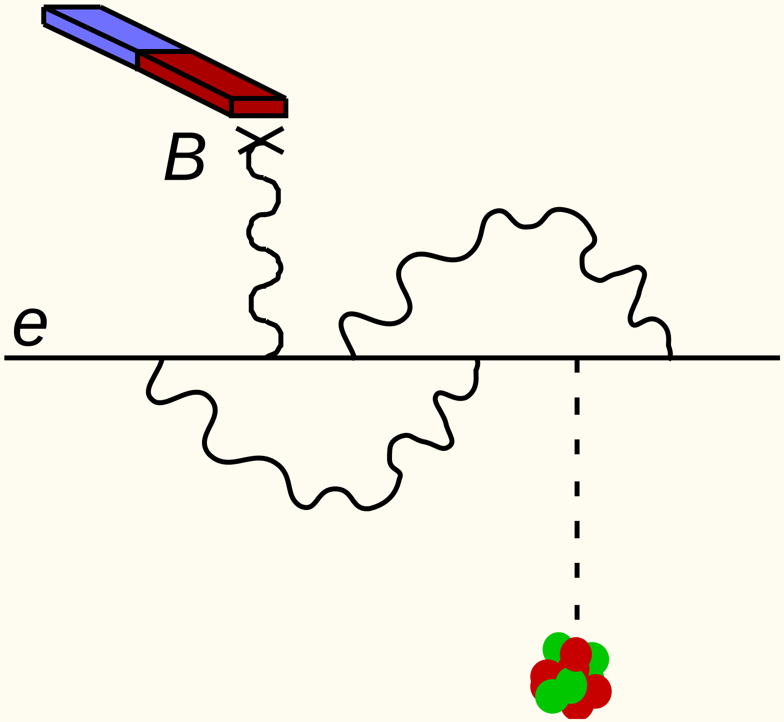
$$+\frac{\alpha}{\pi}\left[1+\frac{(Z\alpha)^2}{6}+(Z\alpha)^4\left(a_{41}\ln\frac{1}{(Z\alpha)^2}+a_{40}\right)+O(Z\alpha)^5\right]$$

$$+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1+\frac{(Z\alpha)^2}{6}\right)+(Z\alpha)^4\left(b_{41}\ln\frac{1}{(Z\alpha)^2}+b_{40}\right)+..\right]$$

two-loop corrections

Main uncertainty

Two-loop determination



<u>Challenges:</u>

Double-counting of lower orders

Subtraction of form-factors (Pachucki)

External fields

Easiest to deal with in dimensional regularization

Results for the structure function

$$Q^{\mu\nu\rho} = \frac{1}{2}\left[\eta\, \mathcal{F}^{\mu\nu\rho} + \xi\, \mathcal{G}^{\mu\nu\rho}\right]$$

$$\mathcal{F}^{\mu\nu\rho} \;=\; q_1^\mu \left(q_1^\rho q_2^\nu - q_1^\nu q_2^\rho\right) + q_1 \cdot q_2 \left(g^{\mu\rho} q_1^\nu - g^{\mu\nu} q_1^\rho\right)$$

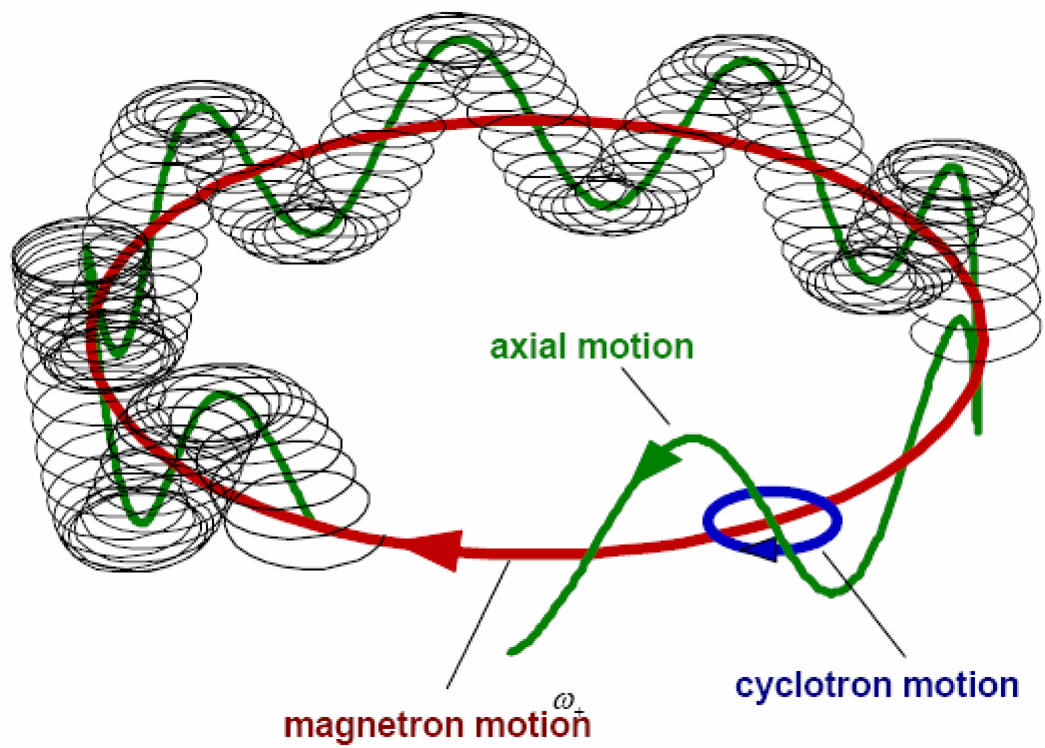
$$\boxed{\begin{aligned}\eta = -\frac{\alpha}{4\pi}\frac{2}{3\varepsilon} + \left(\frac{\alpha}{4\pi}\right)^2 \Bigg[& \left(\frac{2528}{81} - \frac{169}{54}\pi^2\right)_{\mathrm{VP}} \\ & - \frac{283}{10} + \frac{169}{120}\pi^2 - \frac{4}{15}\pi^2 \ln 2 + \frac{2}{5}\zeta(3) - \frac{16}{3\varepsilon}\Bigg]\end{aligned}}$$

Note: divergences 1/ε signal the presence of logarithms

$$\frac{m^\varepsilon}{\varepsilon} - \frac{(Z\alpha m)^\varepsilon}{\varepsilon} = \ln\frac{1}{Z\alpha} + O(\varepsilon)$$

Advantage of dimensional regularization: no spurious scales

Bound-electron g-2: measurement



Spin precession (Larmor) frequency

$$h\nu_L = g \cdot \mu_B \cdot B$$

Cyclotron frequency:

$$h\nu_C = \frac{q}{M} B$$

$$g = 2\frac{\nu_L}{\nu_C}\frac{q}{e}\frac{m}{M}$$

From Werth

Improved m_e (new!)

$$b_{40} = -16.4$$

Pachucki, AC, Jentschura, Yerokhin
2005

Using the Mainz group measurements of ν_L / ν_C we get

$$\boxed{m_e\left({}^{12}C^{5+}\right) = 0.000\ 548\ 579\ 909\ 31(29)_{\mathrm{exp}}\,(1)_{\mathrm{th}}\ u}$$

For comparison, the free-electron mass determination,

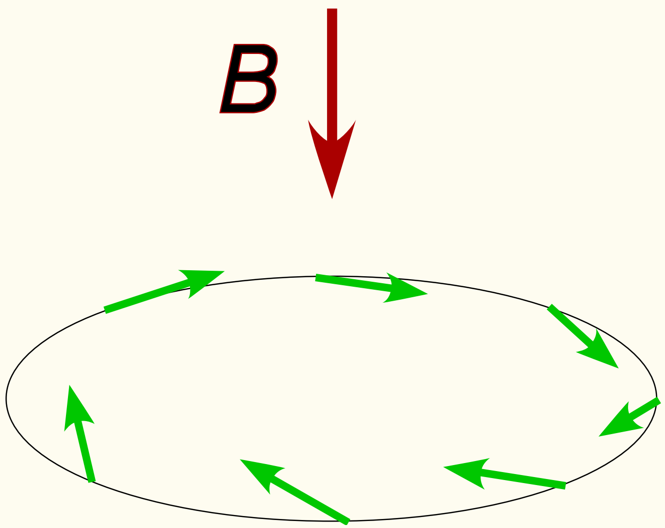
$$m_e\left(\mathrm{free}\right) = 0.000\ 548\ 579\ 911\ 10(120)_{\mathrm{exp}}\ u$$

van Dyck, Farnham, Schwinberg 1995

Muonium and the free muon g-2

How do we determine free muon g-2?

Measure $$\omega_a = \frac{g-2}{2}\frac{e}{m_\mu}B$$



B from NMR: $$\omega_p = \frac{2\mu_p B}{\hbar}$$

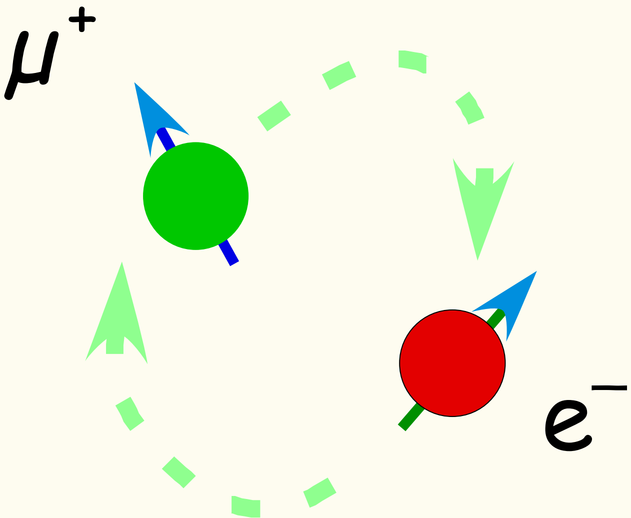
$\dfrac{e}{m_\mu}$ from $$\mu_\mu \equiv g\frac{e\hbar}{4m_\mu}$$

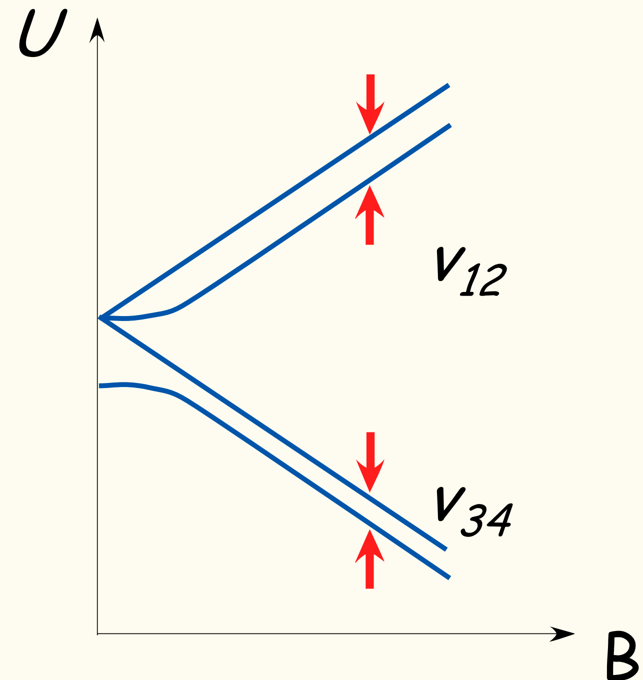
Master formula: $$\frac{g-2}{2} = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

Measured by E821

From muonium

Muonium spectrum determines μ_μ/μ_p





$$\nu_{34} - \nu_{12} \sim \mu_\mu B \quad \Rightarrow \quad \mu_\mu / \mu_p$$

Measured to relative $1.2 \cdot 10^{-7}$ (like $15 \cdot 10^{-11}$ in a_μ)

Will need improvement for the next g-2

Note: preliminary instanton-gas model study (Dorokhov)

$$a_\mu^{\mathrm{LBL}} = 106(10) \cdot 10^{-11}$$ exquisite precision!

Summary

Muonic hydrogen measurement:

Very important for Hydrogen Lamb shift theory
Different nuclei can be studied
Tests of bound-state QED and few-nucleon systems

Muonium HFS: needed for next g-2.

Tests of QED are crucial for our understanding
of bound states and developments in QCD.